ANYONE who constructs a pickup arm has to choose values for the offset angle of the cantilever assembly and the overhang. Anyone mounting a pickup arm must only consider the overhang in order to determine the optimum distance between the pickup arm and turntable pivots. There is a general belief that a pickup arm should be mounted for minimum angular tracking error and that the longer it is, the better. However, the distortion of a given modulation becomes greater as the groove speed decreases, and a pickup arm designed and mounted for minimum tracking error will have its maximum error on the worst possible occasion, namely at the innermost grooves of the record. As the length of an arm increases, its inertia (effective mass) at the stylus tip will usually increase, and this is undesirable.

Two designs are given. The first is for a general purpose record player suitable for 7in, 10in and 12in discs, and the second is for a record player restricted to 7in discs. In both cases, a range of arm lengths is considered, and the most suitable values for the offset angle and overhang are given. Also, for the first design, values of overhang are included for offset angles varying within limits of about ±0.5° from the recommended value. Provided that the offset angle of a pickup arm lies within this range, the mounting may be considered as optimum with the smaller angle slightly favouring 45 rev/min records and the larger angle favouring long-playing records.

The importance of mounting accurately is shown graphically. An error of 0.1in in overhang or 2° in offset angle will more than double the distortion from long-playing records. In view of this, the distances between the stylus and turntable centre, at which the tracking error should be zero, have been given. These distances are independent of the length of the pickup arm and enable the accuracy of the mounting to be checked. A mounting procedure is given for reducing these errors. If the offset angle is fixed and slightly different from the recommended value, an alternative procedure is given for adjusting the overhang for optimum mounting. Neither of these methods requires the offset angle or overhang to be accurately determined. All that is required is an alignment protractor and such a device is obtainable from most hi-fi dealers.

Distortions in reproduction from discs

The quality of reproduction from discs is limited mainly by three factors. Firstly, tracking error, which is the error in alignment between the cantilever assembly and the groove in which the stylus is located. Lateral (horizontal) tracking error is the angle, in the horizontal plane, between the groove, or direction of motion, and the cantilever, and for a conventional pickup arm the lateral tracking error varies with distance from the turntable centre. Vertical tracking error, which only affects stereophonic records, is the difference between the angle which the stylus makes with the vertical when slightly lifted (the arm remaining stationary), and the value specified for the particular record being played. The second factor limiting the quality of reproduction is groove deformation due to the plastic record material being compressed by the stylus tip. However, this distortion tends to reduce the third and most serious form of distortion, tracking distortion due to the cutter and stylus being of a different shape. The flat edge of the V-shaped cutter faces the direction of record motion, and therefore an absence of signal corresponds to a large groove width. As a result of a signal causing the
cutter to move horizontally, as in the case of a monophonic signal or two stereophonic signals approximately equal in amplitude and phase, the groove width becomes narrower causing a stylus of spherical tip to rise. This is the pinch effect. The high-frequency response of a pickup is limited by the radius of the stylus tip since the finite width in the direction of record motion limits the ability to follow groove modulations. The quality of reproduction is reduced further if the tracking mass is insufficient for the stylus to contend with high-frequency velocities (for which a small effective tip mass is favoured) and low-frequency amplitudes (for which a large compliance is favoured).

Elliptical styli

In articles on pickup design, lateral tracking error is usually only briefly considered since the distortion is generally very much less than that resulting from other causes. However, in the last few years, other forms of distortion have been reduced considerably. With pickups of lower effective tip mass and higher compliance, lower tracking masses have been possible. This has enabled styli of smaller radius (0.0005 in and less) to be used with a subsequent reduction in tracing distortion. Tracing distortion has been reduced further by the adoption of elliptical styli, mounted so that the major axes face the direction of record motion. The "vertical" cutting angle has been standardized in the U.S.A. at 15° and it should not be long before this value becomes universal. The result will then be a considerable reduction in vertical tracking error and subsequent distortion when the latest stereophonic records are tracked by the latest pickups, i.e. pickups designed so that a line joining the stylus tip and cantilever pivot is inclined at 15° to the record surface.

Despite their many advantages, the use of elliptical styli results in an increase in distortion from lateral tracking error by causing a time-lag effect. With a spherical tip, the points of contact with the walls of unmodulated grooves and at the crests of waves are always the same as those of the cutter, i.e. a line through these points will be perpendicular to the direction of record motion. However, if the styli is elliptical and tracking error exists, then one point of contact will be slightly ahead of the other. In these circumstances, tracking error cannot be ignored. Even with spherical styli, it is certainly worth spending a few minutes extra in mounting a pickup arm if, as a result, the distortion due to lateral tracking error is reduced to a minimum, perhaps even halved.

Distortion due to tracking error

The main form of distortion due to tracking error is second harmonic and is given by H. G. Baerwald1 as:—

\[ \epsilon = 100 \frac{V}{\mu} \tan \eta \approx 100 \frac{V}{\mu} \eta \]

where \( \epsilon = \% \) 2nd harmonic distortion, \( V_e = \) peak recorded velocity (cm/sec), \( \eta = \) tracking error in radians, and \( \mu = \) groove speed (cm/sec).

Third and higher harmonic distortions amount to less than 10% of this value and are considered negligible. Using the expressions:—

\[ V_{rms} = \sqrt{\frac{V}{2}} = \text{r.m.s. or effective recorded velocity} \]

\[ \text{(cm/sec)} \]


J. K. Stevenson, who is 26, graduated with an honours degree in physics at Birmingham University in 1962 and since then has been at the G.E.C. Hirst Research Centre, Wembley. He is senior member of a team investigating processing techniques for quartz crystals as piezoelectric resonators and determining new synthesis procedures for quartz crystal wave filters. He is a part-time teacher of physics at Willesden Technical College. This article stems from his personal interest in audio equipment.

\[ \phi = \frac{360\gamma}{2\pi} = \text{tracking error (degrees)} \]

\[ x = \text{distance between stylus tip and turntable pivot (in)} \]

\[ s = \text{turntable speed (rev/min)} \]

\[ \mu = (2\pi \times 2.54x) \times \frac{60}{x} \]

and equation 1 may more conveniently be put in the form

\[ \epsilon = 100 \frac{V_{rms} \phi}{2.54 \times 6 \times x} = 9.28 \frac{V_{rms} \phi}{x} \]

A typical maximum value for \( V_{rms} \) may be considered as 10. Therefore,

\[ \epsilon_{max} = 92.8 \frac{\phi}{x} \]

The sign of \( \phi \) (positive or negative) is ignored as it has no effect on the distortion. In order to prevent an excessive groove amplitude in the bass and to improve the signal to noise ratio, a gain of approximately 4 dB per octave with increasing frequency is applied to the recorded sound before it reaches the cutting heads. A reverse characteristic is used for replay. As a result, harmonic distortions are effectively reduced by 4 dB per octave. The effective maximum second harmonic distortion \( \epsilon'_{max} \) is as follows,

\[ \epsilon'_{max} = 10^{-4/10} \epsilon_{max} = 0.631 \epsilon_{max} = 58.5 \frac{\phi}{x} \]

since the variation in pickup voltage is proportional to the recorded velocity. Hence,

\[ \epsilon'_{max} = 1.76 \frac{\phi}{x} \quad \ldots \quad 33 \frac{1}{2} \text{ rev/min} \]

\[ = 1.30 \frac{\phi}{x} \quad \ldots \quad 45 \text{ rev/min} \]

The values for harmonic distortion in this article are for spherical styli: other distortions due to tracking error are considered in Appendix 1 (in the next issue). The important point is that the distortions depend on \( \phi/x \).

Therefore, in a design \( \phi/x \) should be minimized, and not \( \phi \) alone.

Design procedure

Fig. 1 shows tracking errors for a straight pickup arm. By offsetting the pickup the errors are reduced considerably as shown in Fig. 2. Using the symbols in Fig. 3, we wish to determine the optimum values of \( \phi_0 \) and \( f \) when \( x \) lies between given limits. We therefore require a value of \( \theta \) which will remain almost constant as \( x \) varies within this range.

From Fig. 3:—

\[ d^2 = x^2 + l^2 - 2xl \cos \beta = x^2 + l^2 - 2xl \sin \theta \]

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Rearranging, \( 2l \sin \theta = x - \frac{l^2 - d^2}{x} = y \)  
\[ \text{(2)} \]

where \( d = l - f \).

We will consider first the determination of \( \theta_p \) and \( f \) for minimum angular tracking error, and then show how these expressions are modified to reduce the maximum distortion of a given modulation, which is proportional to groove speed and therefore is inversely proportional to \( x \).

**Design for minimum angular tracking error**

A value of \( \theta \) is required which varies least in equation 2 when \( x \) varies between \( x_1 \) and \( x_2 \); \( l \) and \( d \) are constants.

If we plot \( y \) against \( x \) we will obtain a curve which starts at infinity when \( x = 0 \) reduces to a minimum value, and then increases to infinity at \( x = \infty \) as in Fig. 4. The minimum at \( x = x_m \) is given by differentiating equation 2 with respect to \( x \), and equating to zero.

\[
\frac{dy}{dx} = 0 = 1 - \frac{l^2 - d^2}{x^2}
\]

Hence, \( x_m = \sqrt{(l^2 - d^2)} \).

Since \( l \) is given, \( x_m \) is varied by altering \( d \). It is clear that \( x_m \) should lie between \( x_1 \) and \( x_2 \) if we wish the variation in \( 2l \sin \theta \) to be as small as possible for \( x \) varying between \( x_1 \) and \( x_2 \). In particular, the variation will be a minimum if the values of \( y \) at \( x = x_1 \) and \( x = x_2 \) are equal. From equation 2,

\[
x_1 + \frac{l^2 - d^2}{x_1} = x_2 + \frac{l^2 - d^2}{x_2}
\]

Hence, \( x_1 x_2 = l^2 - d^2 = x_m^2 \)

The minimum value of \( y \) then occurs at the geometric mean of \( x_1 \) and \( x_2 \). Using equation 3,

\[
d = (l^2 - x_1 x_2) = l - f
\]

The overhang \( f \) is therefore given by:

\[
f = l - (l^2 - x_1 x_2)
\]

The maximum and minimum values of \( y \) are given from equations 2 and 3.

\[
y_{\text{max}} = x_1 + x_2 = 2l \sin \theta_{\text{max}}
\]

\[
y_{\text{min}} = 2(x_1 x_2)^{\frac{1}{2}} = 2l \sin \theta_{\text{min}}
\]

The design value for \( \theta_p \) is then as follows:

\[
\theta_p = \left[ \sin \left( x_1 x_2 \right)^{\frac{1}{2}} \right] \frac{\sin^{-1} \left( x_1 x_2 \right)^{\frac{1}{2}} + \sin^{-1} x_1 + x_2}{2l}
\]

or \( \sin \left[ \frac{1}{2} \left( \theta_{\text{min}} + \theta_{\text{max}} \right) \right] = \frac{\left( x_1^{\frac{1}{3}} + x_2^{\frac{1}{3}} \right)^3}{4l} \)

**Design for minimum tracking error distortion**

In determining the pickup parameters for minimum lateral tracking error, we minimized the variation in \( 2l \sin \theta \) for \( x \) varying between \( x_1 \) and \( x_2 \) by including a turning point in the curve of \( 2l \sin \theta \) against \( x \) at \( x = (x_1 x_2)^{\frac{1}{3}} \). \( 2l \sin \theta \) then has the same value at \( x_1 \) and \( x_2 \).

In this case, by minimizing the variation in \( \sin \theta \) (i.e., \( \phi \) is as small as possible for \( x_1 \), \( x_2 \)), we automatically minimized the variation in \( \theta_p \) as required. Since \( \phi \) is required to be a very small at the average minimum value of \( x \). Let the tracking error be zero at \( x = x_q \). We have mentioned Fig. 4 which gives \( 2l \sin \theta \) against \( x \). Fig. 5 gives \( 2 \sin \theta / x \) against \( x \) and in addition \( 2 \sin \theta / x \) against \( x \). The distance between these curves at a given
but since \( \sin \theta_0 - \sin \theta_p = 2 \sin \frac{\theta_p - \theta_0}{2} \cos \frac{\theta_p + \theta_0}{2} \)

\[= \frac{\pi}{180} (\theta_0 - \theta_p) \cos \theta_0 \]

and \( \sin \theta_p - \sin \theta_0 = 2 \sin \frac{\theta_p - \theta_0}{2} \cos \frac{\theta_p + \theta_0}{2} \)

\[= \frac{\pi}{180} (\theta_0 - \theta_p) \cos \theta_0 \]

we may replace equation 5 by

\[2l(\sin \theta_0 - \sin \theta_p) = 2l(\sin \theta_p - \sin \theta_0) \cdot \]

This may be put in the form:

\[
x_p = x_0 + x_m^2 \left( \frac{2x_p x_m^2}{x_0^2 + x_m^2} - \frac{1}{2} \left( x_0 + x_m^2 \right) \right)
\]

\[= \frac{2x_p x_m^2}{x_0^2 + x_m^2} - \frac{x_0 + x_m^2}{x_0} \cdot \]

Simplifying,

\[x_m \left( \frac{2x_p}{x_0^2} - \frac{1}{2} \right) + x_m \left( \frac{3x_p}{x_0} \right) = \frac{x_p^3}{x_0} - \frac{x_0^3}{2} = 0. \]

Hence, \( x_m = \frac{x_p^3}{x_0^3} \cdot \frac{828x_0^2 - 4x_p}{x_0} \)

This reduces to:

\[x_m = \frac{0.8284x_0 + 0.1716x_p}{x_0} \cdot \]

\[\text{Fig. 4. Variation of } 2l \sin \theta \text{ with } x \text{ for a pickup arm designed for zero tracking error at } x_0 \text{ and minimum distortion between } x_0 \text{ and } x_p. \text{ As } x_0 < x < x_p \text{ the maximum harmonic distortion } (x_0 \theta - x_p \theta) \text{ decreases to zero, increases to a maximum at } x_p \text{ and then decreases to zero at } x_0. \text{ Note that the largest negative value of } (2l \sin \theta - \sin \theta_p) \text{ occurs at } x_m \text{ and the largest negative value of } 2l(\sin \theta - \sin \theta_p) / x \text{ at } x_p. \]

value of \( x \) is \( 2l(\sin \theta - \sin \theta_p) / x \) which we will minimize since it is approximately proportional to \( (\theta - \theta_p) / x \) and allows a considerable simplification in the algebra.

Both \( 2l \sin \theta \) against \( x \) and \( 2l(\sin \theta - \sin \theta_p) \) against \( x \) have a minimum at \( x_m \), whereas \( 2l(\sin \theta - \sin \theta_p) / x \) against \( x \) has a minimum at a value of \( x \) less than \( x_m \), which we will call \( x_p \). From equations 2 and 3:

\[2l \sin \theta = x + \frac{x_m^2}{x}. \]

Hence,

\[2l(\sin \theta - \sin \theta_p) = \frac{x + x_m^2}{x} - \left( x_0 + x_m^2 \right) = \frac{W}{x} \ldots \]

\( W \) against \( x \) has a minimum at \( x = x_p \). Hence, from Fig. 5, \( x_p \) is the value of \( x \) at which the continuous line is the furthest distance below the interrupted line. Differentiating equation 4 with respect to \( x \) and equating to zero:

\[\frac{dW}{dx} = 0 = \frac{2x_m^2}{x^2} + x_0 + \frac{x_m^2}{x} \cdot \frac{2x_m^2}{x} - \frac{1}{x} \left( x_0 \cdot \frac{x_m^2}{x} \cdot \frac{2x_m^2}{x} \right) \]

\[\therefore \ x_p = \frac{2x_m^2}{x_0} = \frac{2x_m^2}{x_0} \cdot \frac{x_0}{x_m^3} = \frac{x_0 + x_m^2}{x_m^3} \cdot \]

Ideally, we require \( \frac{\theta_0 - \theta_p}{x_p} = \frac{\theta_0 - \theta_p}{x_0} \ldots \]

Fig. 5. Variation of \( 2l \sin \theta / x \) with \( x \). The vertical distance between the lines is proportional to the harmonic distortion of a given modulation.
The design value $\theta_p$ for a given value of $l$ is given from:

$$\sin \theta_p = \sin \theta_0 \frac{1}{2l} \left( x_o + \frac{x_m^2}{x_o} \right) = \frac{1}{l} \frac{x_m^2}{x_p}$$

since $\theta_p = \theta_0$. The overhang $f$ is given by:

$$f = l - \left( 2l - x_m^2 \right)$$

All we require are values for $x_o$ and $x_s$. $x_m^2$ is then determined using equation 6 and the value substituted in equations 7 and 8 to determine $\theta_p$ and $f$ for a given value of $l$.

The value of $x$ less than $x_o$ at which the harmonic distortion (of a given modulation) is the same as at $x_p$ and $x_2$ we will call $x_s$. This is given as follows:

$$\frac{x + \frac{x_m^2}{x}}{x} = \frac{x + \frac{x_m^2}{x_o}}{x_o} = \frac{x + \frac{x_m^2}{x_p}}{x_p} = \frac{x + \frac{x_m^2}{x_2}}{x_2} = \frac{x + \frac{x_m^2}{x_3}}{x_3}$$

From the first two expressions,

$$\frac{1}{x} \left( x + \frac{x_m^2}{x} \right) = \frac{x_o + \frac{x_m^2}{x_o}}{x_o} = \frac{x_s + \frac{x_m^2}{x_s}}{x_s}$$

The two solutions for $x$ are $x_3$ and $x_3$. Hence,

$$x_s = \frac{x_m^2}{x_o}$$

The tracking error changes from positive to negative between $x_o$ and $x_p$. Let it be zero at a value of $x$ which we will call $x_o'$.

$$x + \frac{x_m^2}{x} = \frac{x_o + \frac{x_m^2}{x_o}}{x_o}$$

Rearranging,

$$x^2 - \left( x_o + \frac{x_m^2}{x_o} \right) x + x_m^2 = 0$$

Solutions are $x_o$ and $x_o'$. Hence,

$$x_o + x_o' = x_o + \frac{x_m^2}{x_o}$$

so $x_o' = \frac{x_m^2}{x_o}$, as expected.

Note that whatever design method is used to obtain $\theta_p$ and $f$, the tracking error at a given value of $x$ is given by:

$$\phi = \sin^{-1} \left( \frac{x + f(2l-f)}{2l} \right) - \theta_p$$

i.e. $\phi = \theta - \theta_p$ where $\theta$ is given from equation 2 rewritten in the form:

$$\sin \theta = \frac{x + f(2l-f)}{2l}$$

Equation 9 was used to compare the distortion of a given modulation arising from a pickup mounted as suggested here with that obtained if Bauer's method is used. Results will be given in Appendix II.

**Summary of design formulae**

As $x$ decreases, we have shown that the tracking error changes from positive to negative and back to positive again. The tracking error per unit length is zero at $x_o$ and $x_o'$ and the maximum negative value (at $x_p$) is equal to the positive values at $x_s$ and $x_3$ as indicated in Fig. 5.

The values of $x$ are related as follows:

$$\begin{align*}
\frac{x_o - \frac{x_m^2}{x_o}}{x_3} &= \frac{x_o - \frac{x_m^2}{x_3}}{x_2} \\
\frac{x_o - \frac{x_m^2}{x_3}}{x_2} &= \frac{x_o - \frac{x_m^2}{x_2}}{x_3}
\end{align*}$$

where $x_3 > x_2 > x_3 > x_4 > x_2$.

We must specify two values of $x$ to obtain a solution. If we specify $x_o$ and $x_3$ as in the given designs, then:

$$\begin{align*}
x_o &= \frac{x_m^2}{x_o} \\
x_p &= \frac{2x_o x_3}{x_2 + x_m^2} \\
x_2 &= \frac{x_m^2}{x_o + x_2}
\end{align*}$$

where $x_m^2 = 0.8284x_o + 0.1716x_2$

Alternatively, we can give values for $x_4$ and $x_3$, the limits of $x$ between which we require the tracking error per unit length to be minimized. Then it is easily shown that:

$$\begin{align*}
x_o &= \frac{x_2 x_3}{0.8536 x_3 + 0.1464 x_4} \\
x_o' &= \frac{x_m^2}{x_o} = \frac{x_2 x_3}{0.1464 x_4 + 0.8536 x_o} \\
x_p &= \frac{2x_2 x_3}{x_2 + x_3}
\end{align*}$$

where $x_m^2 = \frac{8x_2^2 x_3^2}{x_2^2 + x_3^2 + 6x_2 x_3}$

The offset angle $\theta_p$ and overhang $f$ for minimum tracking error per unit length are then as follows:—

$$\sin \theta_p = \frac{1}{l} \frac{x_m^2}{x_p}$$

$$f = l - (l^2 - x_m^2)$$

To be concluded.

**Acknowledgement** is due to Alfred Imhof Ltd. for the use of their facilities in the production of our front cover.

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PICKUP ARM DESIGN—2

DESIGN AND MOUNTING OF ARMS FOR MINIMUM DISTORTION DUE TO LATERAL TRACKING ERROR

By J. K. STEVENSON,
B.Sc., Grad.Inst.P., Grad.I.E.E.

Concluded from page 218 of May 1966 Issue

RECORD players may be considered as being of two types, those suitable for all diameter records, and those suitable for only 7in records. Design values are now given for both types.

Design values for 7in discs

x was measured at the start and finish of a number of records of different makes and the values obtained are given in Table 1. \( x_{\text{outer}} \) which was measured for 7in and 12in discs, was found to be fairly constant and varied at the most by 1/32in. The values for \( x_{\text{inner}} \) are representative values for the minimum distance from the record centre and \( x_{\text{min}} \) denotes the minimum distance in exceptional cases, being the minimum value obtained for \( x_{\text{inner}} \) from measurements on a batch of records of different makes and different types of music.

Pickup arms have been designed using equations 6, 7, and 8. In the design of an arm restricted to 7in records, \( x_0 \) and \( x_2 \) were chosen as follows.

\[
x_0 = x_{\text{inner}} = 2.125\text{in}
\]

\[
x_2 = x_{\text{outer}} = 3.281\text{in}
\]

\( x_0, x_2, x_p, x_g \) and \( x_2 \) were then determined and the values are given in Table 2. Maximum distortion of a given modulation occurs at the start of a record, and also at \( x = 2.49\text{in} \), and \( x = 2.00\text{in} \). The distortion becomes zero at \( x = 3.00\text{in} \) and \( x = 2.13\text{in} \). In this design, the distortion is set to zero at the normal finish of a 7in disc (a desirable feature) and increases to the maximum value at \( x = 2.00\text{in} \). However, it is unusual for the modulated section of a 7in disc to continue as far as \( x = 2\text{in} \).

Design values for all discs

In the case of a record player suitable for 7in, 10in and 12in discs, the situation is more involved. In considering the distortions from 7in discs, it must be remembered that the turntable speed for these records is different to that for 10in and 12in discs. Three designs were obtained as shown in Fig. 6, and as the offset angle and overlap vary fairly linearly between them, the extreme designs 1A and 1C provide limits for overlap and offset angle between which any value of either of these parameters may be chosen, and the value for the other immediately given. With the smaller offset angle (design 1C), the maximum harmonic distortion for 7in discs tracking at 45 rev/min is less than that for long-playing records, as seen from Fig. 7. As the angle increases,


![In part 1, the author maintained that whilst distortions in disc reproduction are gradually being reduced, tracking error distortion has not been. It was pointed out that it is possible to design and mount an arm so that distortion due to lateral tracking error is less than 1%.

After discussion of distortion due to tracking error, design formulae are derived for offset angle and overlap for minimum tracking error distortion.

In part 2, two designs are presented, one for 7in discs and one for 7, 10 and 12in discs. Tracking error is shown to be critically dependant on mounting errors, and in view of this, the author outlines two mounting procedures. Pickup arm shape and optimum tracking mass are also considered.](image)

TABLE 1

<table>
<thead>
<tr>
<th>x0</th>
<th>x1</th>
<th>Discs considered</th>
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</thead>
<tbody>
<tr>
<td>2.1</td>
<td>2.2</td>
<td>10&quot;, 12&quot; (33) rev/min</td>
</tr>
<tr>
<td>2.2</td>
<td>2.4</td>
<td>7&quot; (45 rev/min)</td>
</tr>
</tbody>
</table>

TABLE 2

<table>
<thead>
<tr>
<th>Design</th>
<th>x3</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x9</th>
<th>x10</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>2.375</td>
<td>2.794</td>
<td>3.660</td>
<td>3.269</td>
<td>3.160</td>
<td>3.422</td>
<td>7&quot;</td>
</tr>
<tr>
<td>1C</td>
<td>2.200</td>
<td>2.512</td>
<td>3.412</td>
<td>3.232</td>
<td>3.190</td>
<td>3.190</td>
<td>7&quot;</td>
</tr>
<tr>
<td>2</td>
<td>2.125</td>
<td>3.281</td>
<td>3.300</td>
<td>3.004</td>
<td>2.488</td>
<td>2.335</td>
<td>7&quot;</td>
</tr>
</tbody>
</table>

In a design, the distortion of a given modulation is maximum at the largest value of \( x \) \( (x_3) \) and set to zero at \( x_p \). Then:

\[
x_0, x_2, x_p, x_3 \text{ are the values at which the distortion is zero and } x_0, x_2, x_3 \text{ are the values at which the maximum is set.}
\]

\[
x_p > x_0, x_p > x_0, x_p > x_0 \text{ as shown in Fig. 5 (last month).}
\]
then provided the overhang is adjusted accordingly, the maximum distortion for long-playing records gradually reduces, and the maximum distortion for 7in discs increases. For the larger offset angle (design 1A), the maximum distortion for 7in discs is about 2½ times as great as that for long-playing records. However, it is still less than 2½ second harmonic even for an 8in pickup, and in view of the fact that other forms of distortion will be lower for 7in discs at 45 rev/min than for 10in or 12in discs at 33 1/3 rev/min, the design is still suitable. The values used in these designs are given in Table 2. Note that for designs 1A to 1C the tracking error changes from positive to negative for a long-playing record, and changes from negative to positive for a 7in disc. If we had disregarded 7in discs and obtained a design with minimum distortion for long-playing records, i.e., one for which the distortion changes from positive to negative and back to positive again, the improvement would be very small but the distortion from 7in discs would by comparison be enormous.

The intermediate design 1B is recommended although, as mentioned earlier, provided that the offset angle lies within the given range, the mounting may be considered as optimum with the smaller angle slightly favouring 45 rev/min records and the larger angle favouring long-playing records.

45 rev/min records are slightly favoured insofar as most forms of distortion are inversely proportional to the groove speed or a power of the groove speed i.e.,

distortion \propto \left(\frac{1}{v}\right)^n \propto \left(\frac{1}{v^2}\right)^n, \text{ where } n \geq 1.

The maximum distortion of a given modulation (the total distortion quoted correspond to a 10 cm/s r.m.s. recorded velocity. Of the three designs 1A, 1B and 1C, 1B is recommended. The slightly greater harmonic distortion for 7in discs is counteracted by a reduction in distortion from other causes, as a result of the faster turntable speed. For an 8in pickup arm mounted as suggested, the distance between the edge of a 12in disc and the centre of the arm pivot is 1 ½ in. A shorter distance corresponding to a shorter arm is impractical if adequate compensation is to be made for side-thrust.

Fig. 6. Design values. The values of distortion quoted correspond to a 10 cm/s r.m.s. recorded velocity. Of the three designs 1A, 1B and 1C, 1B is recommended. The slightly greater harmonic distortion for 7in discs is counteracted by a reduction in distortion from other causes, as a result of the faster turntable speed. For an 8in pickup arm mounted as suggested, the distance between the edge of a 12in disc and the centre of the arm pivot is 1 ½ in. A shorter distance corresponding to a shorter arm is impractical if adequate compensation is to be made for side-thrust.

Fig. 7. Variation of 2nd harmonic distortion with x. The values of distortion are typical maximum values, as distinct from occasional peak values due to exceptionally heavy modulation.

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variable by about 1°. If the head is mounted to the arm with two nuts and bolts, then one of the holes need only be made fractionally wider. Unless the fit is very close, such a movement may already be possible. Clearly it is best to check the mounting as given below before widening any holes. Values of x are given to three decimal places for the benefit of precision engineers. Using an alignment protractor and a metal rule, the average constructor should be able to measure x to ±0.001in without undue difficulty.

(i) Set x to 2.375in and adjust the overhang by moving the pickup arm pivot towards or away from the turntable centre, until the tracking error is zero.

(ii) Set x to 4.606in (4.6in) and observe whether the tracking error is positive or negative. If it is uncertain which is which, set x to 2.5in and the indicated tracking error is positive.

(iii) If the tracking error at x = 4.606in is positive, then both the offset and overhang are too small: the offset angle should be increased slightly. Similarly, if the tracking error is negative, then both the offset and overhang are too large: the offset angle should be decreased slightly.

The steps (i) to (iii) are then repeated until the tracking errors at both values of x are negligible (less than 1°).

(b) Alternative mounting (design between 1A and 1C)

Although 1B is considered by the writer to be the best design, any design between 1A and 1C may be considered as good and will be optimum insofar as no other method of mounting can result in lower distortion (as a result of lateral tracking error) from both long-playing and 45 rev/min records. As seen from Fig. 7, design 1A favours long-playing records and design 1C favours 45 rev/min 7in discs.

It may not be possible considered, or it may be felt undesirable, to alter the offset angle, particularly if different pickups are used in the same arm. In this case, provided that the offset angle for a given value of i is within the range of values in Fig. 6, the following method should be used:

(i) Set x to 2.375in (x1 for design 1B), and adjust the overhang until the tracking error is zero.

(ii) Set x to 4.606in (x2 for design 1B), and observe whether the tracking error is positive or negative.

(iii) If positive, then both the offset and overhang are too small for this design: move towards 1C which requires a smaller offset angle and overhang. If negative, move towards 1A.

(iv) To move towards 1C, x is set to a smaller value of x, and f is reduced until φ = 0. From Fig. 10, the value of x*, corresponding to the new x, is obtained. x is then set to the new x* (smaller than before) and the tracking error observed. To move towards 1A, a larger value for x, is chosen and φ set to zero as before. The new x* is given from Fig. 10 and the tracking error observed.

(v) Return to stage (iii) and repeat until the tracking error at x* is negligible (less than 1°).

If φp for a commercial pickup arm is greater than the 1A value, it has most likely been designed for minimum tracking error. The mounting can be improved by using a value of f slightly smaller than the manufacturer's suggested value so as to reduce the large positive tracking error at the inner grooves. An alignment protractor should be used and φx at x = 2.5in reduced until equal in magnitude to the largest negative value of φx which will occur between 2in and 4in. If φp is less than the 1C value, allowance has probably not been made for the faster turntable speed of 7in discs in which case, provided that φp is less than 1° from the 1C value, Fig. 10 may be used with the line slightly extended. Otherwise φx at x = 2.5in (φ/2.125) should be adjusted until equal to the value of 5.75in (φ/5.7). Both should then be greater than the maximum value occurring between 2in and 4in.

Therefore, if a commercial pickup arm is to be mounted and the recommended methods given earlier are not used, φx at x = 2.5in should be adjusted by varying f to equal the other maximum value between 2in and 5.75in. This will occur between 2in and 4in if φp is too large and at 5.75in if φp is too small.

Shape of pickup arm

Fig. 11a gives the shape of a conventional pickup arm. In order to reduce friction, the most suitable method of mounting is a single pivot (unipivot). A line joining the pivot and stylus tip should be horizontal otherwise the pickup cannot move vertically upwards when tracking a warped record. The centre of gravity of the assembly must lie below this line for stability and directly beneath it with the pickup untitled.

A means of lateral adjustment is usually provided and, in general, this consists of a device clamped to the arm at the pivot, being movable in the direction AA so as to apply a moment to the arm to remove any tilt. If a larger tracking mass is required, the counterbalance, or part of it, is moved towards the pivot. This decreases the anti-clockwise moment due to the counterbalance as seen from B, and a pickup which was previously correctly adjusted will tilt slightly in a clockwise direction causing a stylus tip to press on the inner wall of a record groove.

A suggested shape for a pickup arm is given in Fig. 11b. With the centre of gravity always lying along the length of the arm, lateral adjustment is no longer necessary when altering the tracking mass. Also, the pickup arm is slightly shorter for a given value of l, and will therefore be lighter.

Further advantage is that the lateral adjustment at...
reduced by using a heavier counterbalance nearer the pivot to reduce \( l \). However, the mass of the pickup assembly is given by

\[
M_1 + M_2 - M_3 = M_1 \left( 1 + \frac{l}{l_e} \right) + M_2 \left( 1 + \frac{l}{l_e} \right)
\]

and \( M_3 \) should not be too great otherwise unnecessary friction and wear are liable to occur in the pivot bearings.

**Tracking mass for optimum reproduction with minimum record wear**

The tracking mass, \( M_t \), must be sufficient for the stylus to remain permanently in contact with the groove walls. At low frequencies we require the compliance of the stylus suspension, in particular the lateral compliance, \( C_t \), to be as large as possible to enable the stylus to cope with modulations of large amplitude. At high frequencies we require the effective tip mass, \( M_{etm} \) (the inertia of the stylus and elastic support at the stylus tip) to be as small as possible so that the force on the stylus resulting from the considerable accelerations is much less than that due to the tracking mass. Also, we require the inertia of the pickup assembly, \( M_p \), to be as small as possible so that forces on the stylus caused by tracking warped records are reduced to a minimum.*

To enable the stylus to follow the most difficult modulations, it is suggested that the following should be satisfied

\[
M_t \geq M_t'
\]

where

\[
M_t' = \frac{5}{10^5 C_t} + 1000 M_{etm} + \frac{M_p}{40} \quad \ldots \quad 10
\]

\( M \) is given in gm and \( C \) in cm/dyne, e.g. if the compliance is 15,10^{-5} cm/dyne the effective tip mass \( \frac{1}{2} \) mg, and the inertia of the pickup assembly 13\frac{1}{2} gm, the minimum tracking mass is 1 gm.

If the tracking mass is smaller than \( M_{t0} \) then for heavily modulated grooves the contact between the stylus and the groove walls is liable to be intermittent, damaging the walls in addition to increasing the noise content of the resulting signal.

Usually the vertical compliance, \( C_v \), need not be considered, for although the value may be as low as half the lateral compliance, the vertical groove modulation only results from the difference (in amplitude and phase) between the two stereophonic signals; such a modulation seldom exceeds half the lateral modulation especially at low frequencies where the phase differences are normally very small. However, if for a stereophonic pickup \( C_v < \frac{1}{2} C_t \), 2\( C_t \) should replace \( C_t \) in equation 10. Even a monophonic pickup requires a vertical compliance to track the modulation resulting from the pinchoff effect. The second harmonic vertical modulation due to a spherical stylus tracking a laterally cut groove is likely to reach 20%\;\text{in}. In this case we require \( C_v < 1/5C_t \), for a monophonic pickup, otherwise \( 5C_t \) should replace \( C_t \) in equation 10.

Unfortunately, the maximum lateral displacement of the stylus occurs at the same instant as a maximum vertical displacement due to the pinchoff effect. As a result, it appears preferable for \( 5/10^5 C_t \) in equation 10 to be replaced by

\[
\frac{5}{10^5 C_t} \left( 1 + \frac{C_v^2}{25C_t^2} \right)^{1/2}
\]

However, the difference is usually very small except for

* Normally, a faster groove speed is an advantage; this is one of the rare instances where the converse is true.

---

**Inertia of pickup assembly**

A pickup arm should be constructed so that its inertia or effective mass at the stylus tip is as small as possible.† The inertia of the pickup assembly at the stylus tip is given by the moment of inertia of the pickup arm and counterbalance about the axis of rotation (the arm pivot) divided by the square of the effective arm length i.e.

\[
M = M + M_d + \frac{1}{l} M_d^2
\]

as given in Fig. 11b where \( M_d, M_2, M_3 \), and \( M_4 \) denote the inertia of the pickup assembly and masses of the pickup (including the mounting shell), arm, and counterbalance, respectively. The mass of the arm is considered to be evenly distributed along the length and, for simplicity, \( M_3 \) is assumed to be concentrated at its centre of gravity, and \( M_4 \) concentrated at the stylus tip. Taking moments about the pivot with the pickup balanced,

\[
M_d + \frac{1}{l} M_d^2 = M_d
\]

Hence, the mass of the counterbalance is given by

\[
M_d = (M_1 + M_2) l_e
\]

and the inertia as follows

\[
M_d = M_1 l_e^2 + \frac{1}{l} M_2 l_e^2
\]

† See equation 10 and Appendix III.
monophonic pickups from a designer who has disregarded vertical compliance.

Equation 10 is obtained from the following

\[ M_t' \delta = \frac{z}{C_t} + M_{stim} \delta_s + M_{\rho s} \delta \]

where \( z, \delta_s, \) and \( \delta \) denote the maximum displacement of the stylus, the maximum acceleration of the stylus tip, and the maximum deformation of the arm, respectively. Representative values\(^8\) are \( z = 0.005 \text{ cm}, \delta_s = 1000g, \delta = 0.025g, \) where \( g = 980 \text{ cm/sec}\(^2\). Clearly, the downward force due to the stylus mass should be greater than the sum of these three forces which try to prevent the stylus remaining in perfect contact with the groove walls. The forces resulting from resistive damping are by comparison very small for modern arms and have not been included: this is permissible as we have considered the worst possible case, although it is most unlikely that all these forces will be maximum at the same time. Equation 11 is based on three equations given by Professor F. V. Hunt\(^4\) who divided the downward force into three equal parts, requiring each to equal or exceed the three individual forces on the right of equation 11 (with the force due to arm damping added to \( M_{\rho s} \)). This constraint, although useful for determining suitable design targets, is unnecessary. Note that side-thrust can amount to as much as 20% of the downward pressure so that the maximum lateral deformation due to side-thrust, \( z_t \), is given by \( z_t = 15 \cdot 10^5 M_{tg} C_t \). Hence, if the lateral compliance of the stylus suspension is very large in relation to the effective tip mass, compensation for side-thrust is especially important to avoid large stylus deflections.

An important point to consider is that a stylus cannot track frequencies above the stylus-groove resonant frequency, \( f_{rs} \), where

\[ f_{rs} = \frac{1}{\pi \sqrt{\frac{C_p M_t}{C_s}}} \]

where \( r \) is the radius of the stylus tip (\( in \)); a value of 3.76, \( 10^{15} \text{ dynes/cm}^2 \) has been used for the plane-stress elastic modulus of the vinylite record material. It is necessary for the stylus to be able to track the highest frequencies which occur on a record otherwise the record will be permanently damaged. With an upper audio limit of 15 kc/s, \( f_{rs} \) must not be less than 30 kc/s, the second harmonic of a lateral signal of 15 kc/s; 30 kc/s occurs as a vertical pinch effect modulation. Hence, \( M_{stim} \leq 0.00693 \cdot 15^{1/2} \). To track at 3 gm with styli of 0.001in, 0.0007in and 0.0005in, the effective tip mass must not exceed 1.00, 0.89 and 0.79 mg respectively; tracking at 1 gm, these values become 0.69, 0.62 and 0.55 mg.

The stylus-groove resonance is usually sufficiently excited to introduce audible noise by intermodulation. Scanning loss, the high frequency loss due to the finite size of the stylus in relation to the groove modulations, will prevent this excitation if \( f_{rs} \) is sufficiently large. The condition which must be satisfied is \( M_{stim} \leq 0.197r M_t \). Hence, to track at 3 gm or 1 gm with styli of 0.001in, 0.0007in, and 0.0005in, the effective tip mass for a 'low noise' signal must not exceed 0.59, 0.41, 0.30 mg, and 0.20, 0.14, and 0.10 mg respectively.

To summarise, \( M_t \) should satisfy the following conditions:

\[ M_t \geq \frac{5}{10^5 C_t} + 1000 M_{stim} + \frac{M_p}{40} \]

\[ M_t \geq \frac{k}{10^5 C_t} + 1000 M_{stim} + \frac{M_p}{40} \]

\[ M_t \geq \frac{3.10^3 M_p}{r} \]

where \( k = 2.5 \) for a stereophonic pickup and \( 1 \) for a monophonic pickup. Also, for a low noise signal,

\[ M_t \geq \frac{5.1 M_{stim}}{r} \]

If the above conditions allow, \( M_t \) should be set to a value within the range 1-3 gm if \( r = 0.0005 \text{ in} \) or 1-4 gm if \( r \geq 0.0007 \text{ in} \). With \( M_t \) less than 1 gm, dust in record grooves becomes a tracking problem: if greater than the upper limit, record wear will occur as a result of the groove deformations no longer being within the elastic limit or the record material. Much useful advice on pickups is given in a recent book by J. Walton\(^4\) who stresses the importance of a low effective tip mass, the most important consideration when choosing a pickup.

To conclude, there is no practical advantage in a properly mounted pickup arm being longer than 9in or 10in. A 12in arm as well as being heavier usually has a larger inertia at the stylus tip: it also requires more space for mounting. The importance of mounting accurately is usually underestimated. The distortion from a pickup with a 12in arm and an error in mounting of \( \pm 1/20 \text{ in} \) is greater than the distortion from a correctly mounted 9in pickup. In view of this, an alignment protractor should be considered as essential when mounting a pickup.

**APPENDIX**

According to E. R. Madsen\(^5\), intermodulation in a lateral cutting appears as modulation of the even harmonics, and in a vertical cutting as modulation of the odd harmonics. The percentage distortion, \( \varepsilon_{rms} \), due to an incorrect vertical tracking error is given by

\[ \varepsilon_{rms} = 100 \left[ \frac{\cos (\alpha - \phi) - \cos (\alpha + \phi)}{\cos (\alpha - \phi) + \cos (\alpha + \phi)} \right] \]

where \( \tan \sigma = \sqrt{2 \cdot \frac{V_{rms}}{V_x}} = 5.32 \cdot \frac{V_{rms}}{V_x} \)

Therefore, when as in our case \( \phi < 5^\circ \), this expression for distortion may be reduced to

\[ \varepsilon_{rms} = 200 \sin \phi \tan \alpha \approx 200 \left( \frac{\pi}{180} \right) \sin \phi \tan \alpha = 18.6 \cdot \frac{V_{rms} \phi}{V_x} \]

Note that the distortion is proportional to \( \phi \). Distortion due to lateral tracking error is similarly proportional to \( \phi \).

If an elliptical stylus is used, lateral tracking error will cause the stylus to sink slightly further into an unmodulated groove. It can be shown\(^6\) that

\[ \varphi = 2a \left( 1 - \frac{b^2}{a^2} \right) \sin \phi \approx 2a \left( 1 - \frac{b^2}{a^2} \right) \sin \phi \approx \frac{\pi \left( a - \frac{b^2}{a^2} \right)}{90} \cdot \phi \]

where \( a \) and \( b \) denote the major and minor radii, respectively.


\(^3\) Private communication to C. Dineen, July 1965.
of the horizontal cross-section through the points of contact with the groove walls, and \( p \) the distance in the direction of record motion between these points of contact. The distortion, peculiar to elliptical styli, due to the points of contact not being perpendicular to the groove walls, depends on the time difference, \( t \), i.e. the time taken for the groove to move a distance \( p \).

\[
t = \frac{2a^2}{3a} \int \frac{d\phi}{\sqrt{a^2 - b^2}}
\]

where \( a, b \), and \( p \) are in inches and \( t \) in seconds. The distortion depends on \( \phi/t \) and may be reduced by reducing this quantity.

When tracking with styli of both circular and elliptical cross section, it is therefore evident that the maximum distortion due to tracking error is least when \( \phi/t \) is a minimum, as in the given designs.

**APPENDIX II**

An examination of pickup arm manufacturers' recommended values of offset angle and overhang reveals considerable discrepancies. Some manufacturers have clearly determined these values on a trial and error basis: others have minimised the angular tracking error forgetting (or not knowing) that the distortion resulting from tracking error is inversely proportional to the distance from the turntable centre. The most suitable values appear to have been obtained from a graph by Bauer who (using our notation) derived the following formulae.

\[
\begin{align*}
\phi &= 57.3 \times x_2 \left(1 + \frac{x_2}{x_4}\right) \\
f &= 2 \frac{x_2^3}{x_4} \left(1 + \frac{x_2}{x_4}\right)^3 + \frac{x_4}{x_2}
\end{align*}
\]

Bauer set \( x_2 \) and \( x_4 \) to the radii of the inner and outer grooves, and therefore \( x \) is maximum at the extreme limits of the stylus movement. Bauer's values are \( x_2 = 5.719\text{in}, x_4 = 1.751\text{in} \).

Although suitable at the time, \( x_2 \) is too small for a modern record player. The tracking error which changes from positive to negative and back to positive between \( x_2 \) and \( x_4 \) is still negative when \( x = 2.1\text{in} \). Replacing these constants with the values used in the present design 1C (\( x_2 = 5.719\text{in}, x_4 = 2.038\text{in} \)) reduces the maximum distortion due to lateral tracking error of a given modulation by 16% for an 8in pickup and 19% for a 10in pickup. However, unlike the present design, the minimum of \( x \) is less than the values \( x_2 \) and \( x_4 \). If our design DC is used, the improvements become 20% and 27%, respectively. Bauer made two approximations: in an expression for \( \sin^2\theta \) (21 - \( f \)) is replaced by \( 2\sin\theta \), and \( \sin^2\theta \) itself is replaced by \( \theta/180 \). These approximations have not been used in the present analysis which, as a result, is more extensive. The reduction in distortion is fairly small and it may not be considered worthwhile to modify an existing pickup arm. However, if a new arm is being designed, values of offset angle and overhang have to be chosen, and as no extra work is involved in using the values given here in preference to Bauer's, it would be foolish to disregard these improved values on the grounds that the reduction in distortion is very small. It is a step in the right direction although two or three such steps may have to be made before the improvement is audible.

**APPENDIX III**

The movement of a stylus relative to a pickup produces the required electrical signals. If the stylus moves very slowly from side to side (or up and down), the pickup will follow these movements and result in a negligible signal. Maximum signal is obtained at the transition frequency \( f_{tr} \), the frequency of arm resonance given by:

\[
f_{tr} = \frac{1}{2\pi \sqrt{M_s/C}}
\]

where \( M_s \) is the inertia of the pickup assembly and \( C \) the compliance (or inverse stiffness) of the stylus suspension.

We require the lateral arm resonant frequency to be greater than 1 c/s so that the effect of the stylus moving towards and away from the turntable centre as a result of eccentric record-mounting holes is negligible. Similarly, for stereophonic pickups we require the vertical arm resonant frequency to be greater than about 10 c/s so that signals are not obtained from ripples and warps. Although sub-audio, these signals are liable to overload the amplifier. Both resonant frequencies should be less than 20 c/s to permit a flat frequency response down to 30 c/s.

The optimum vertical resonant frequency is about 15 c/s. A stereophonic pickup with a vertical compliance of 5.1 x 10^-6 cm/dyne restricts the inertia of the pickup assembly to the range 13 to 51 gm corresponding to resonant frequencies of 20 and 10 c/s respectively (an easy requirement). However, many modern pickups have a compliance of 10.1 x 10^-6 cm/dyne corresponding to an inertia of 6 to 25 gm and some pickups require an even smaller inertia. Since the pickup is liable to weigh about 10 gm, it is evident that the effective mass of a present-day arm (and counterbalance) as seen at the stylus tip should be as small as possible; this is also suggested by equation 10.

The lateral compliance is usually equal or up to twice as large as the vertical compliance so that the lateral resonance occurs at the same or a slightly lower frequency than the vertical resonance. Since the lower limit for lateral resonance is 1 c/s, it is clear that if the frequency of vertical resonance is suitably fixed, the lateral resonant frequency requirement is automatically satisfied. In these circumstances, only a small amount of damping is necessary. Any additional damping required should be associated with the arm pivot and not the stylus suspension and preferably of a viscous type.

If a pickup which has been lowered until the stylus just touches a record is released, it will sink slightly lower at the same time compressing the stylus suspension. The compliance is a measure of the 'springiness' of this suspension and denotes the distance relative to a stationary pickup that the stylus will move as a result of a given force acting on it. A larger compliance implies that the stylus can move more easily relative to the pickup, and therefore the force on the stylus tip due to a groove modulation is smaller. Hence the minimum tracking mass is smaller. The stylus is driven by the groove laterally and upwards and relies on the vertical compliance of the stylus suspension and gravity for downward movement; vertical compliance above the vertical arm resonant frequency and gravity below this frequency. The downward force resulting from the tracking mass becomes the force on the effective tip mass due to the vertical compliance of the stylus suspension acting as a spring. Therefore, the smaller the effective tip mass, the smaller the force required by the stylus to follow a high frequency modulation requiring a large acceleration. Hence the minimum tracking mass is smaller.

**Correction:** We regret the error which occurred on p.216 of the May issue. The 20th line from the bottom of the second column should start with \( \sin^2\theta = \sin^2\theta \), not \( \sin\theta \). In the caption to Fig. 4, \( x_2 < x_4 \) was used, but the caption should have conveyed "as \( x \) varies from \( x_2 \) to \( x_4 \) . . . . . ." On the left hand side of the first equation on p.217 (second column): \( \theta_0 \) should have read \( \theta_0 \). But since \( \theta_0 \), which corresponds to \( x_0 \), is equal to \( \theta_0 \), the design value, \( \theta_0 \) may be taken as \( \theta_0 \).